Sequences and Series

Algebra 2
Chapter 12

- This Slideshow was developed to accompany the textbook
  - *Larson Algebra 2*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

Slides created by
Richard Wright, Andrews Academy
rwright@andrews.edu

12.1 Define and Use Sequences and Series

- Sequence
  - Function whose domain are integers
  - List of numbers that follow a rule
  - 2, 4, 6, 8, 10
    - Finite
  - 2, 4, 6, 8, 10, …
    - Infinite

12.1 Define and Use Sequences and Series

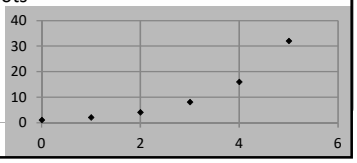
- Rule
  - $a_n = 2n$
- Domain: (n)
  - Term's location (1st, 2nd, 3rd…)
- Range: (a_n)
  - Term's value (2, 4, 6, 8…)

12.1 Define and Use Sequences and Series

- Writing rules for sequences
  - Look for patterns
  - Guess-and-check
- $\frac{2}{5}, \frac{2}{25}, \frac{2}{125}, \frac{2}{625}, \ldots$
- 3, 5, 7, 9, …

12.1 Define and Use Sequences and Series

- To graph
  - n is like x; a_n is like y
  - The graph will be dots
    - Do NOT connect the dots



12.1 Define and Use Sequences and Series

- Series
  - Sum of a sequence
  - 2, 4, 6, 8, … → sequence
  - 2 + 4 + 6 + 8 + · · · → series

12.1 Define and Use Sequences and Series

- Sigma notation
  - Finite

$$2 + 4 + 6 + 8 = \sum_{i=1}^{4} 2i$$

Upper limit

Lower limit

Index of summation (variable)

  - Infinite

$$2 + 4 + 6 + 8 + \cdots = \sum_{i=1}^{\infty} 2i$$

12.1 Define and Use Sequences and Series

- Write as a summation
  - $4 + 8 + 12 + \cdots + 100$
- $2 + \frac{3}{4} + \frac{4}{9} + \frac{5}{16} + \cdots$

12.1 Define and Use Sequences and Series

◈ Find the sum of the series

$$\sum_{k=5}^{10} k^2 + 1$$

12.1 Define and Use Sequences and Series

◈ Some shortcut formulas

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

12.1 Define and Use Sequences and Series

◈ Find the sum of the series

$$\sum_{k=1}^{10} 3k^2 + 2$$

12.2 Analyze Arithmetic Sequences and Series

- Arithmetic Sequences
  - Common difference (d) between successive terms
    - Add the same number each time
  - 3, 6, 9, 12, 15, …
    - d = 3

- Is it arithmetic?
  - -10, -6, -2, 0, 2, 6, 10, …
  - 5, 11, 17, 23, 29, …

12.2 Analyze Arithmetic Sequences and Series

- Formula for n^{th} term
  - $a_n = a_1 + (n - 1)d$

- Write a rule for the n^{th} term
  - 32, 47, 62, 77, …

12.2 Analyze Arithmetic Sequences and Series

- One term of an arithmetic sequence is $a_8 = 50$. The common difference is 0.25. Write the rule for the n^{th} term.

12.2 Analyze Arithmetic Sequences and Series

- Two terms of an arithmetic sequence are $a_5 = 10$ and $a_{30} = 110$. Write a rule for the n^{th} term.

12.2 Analyze Arithmetic Sequences and Series

- Sum of a finite arithmetic series
  - 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10
  - Rewrite
    - 1 + 2 + 3 + 4 + 5
    - 10 + 9 + 8 + 7 + 6
    - 11+11 +11+11 +11 = 5(11) = 55
  - Formula
    - $S_n = n\left(\frac{a_1+a_n}{2}\right)$

12.2 Analyze Arithmetic Sequences and Series

- Consider the arithmetic series
  - 20 + 18 + 16 + 14 + ⋯
- Find the sum of the first 25 terms.

12.2 Analyze Arithmetic Sequences and Series

- Consider the arithmetic series
  - $20 + 18 + 16 + 14 + \cdots$
- Find n such that $S_n = -760$

12.3 Analyze Geometric Sequences and Series

- Created by multiplying by a common ratio (r)
- Are these geometric sequences?
  - 1, 2, 6, 24, 120, …
  - 81, 27, 9, 3, 1, …

12.3 Analyze Geometric Sequences and Series

- Formula for n^{th} term
  - $a_n = a_1 \cdot r^{n-1}$
- Write a rule for the n^{th} term and find a_8.
  - 5, 2, 0.8, 0.32, …

12.3 Analyze Geometric Sequences and Series

- One term of a geometric sequence is $a_4 = 3$ and $r = 3$. Write the rule for the n^{th} term.

12.3 Analyze Geometric Sequences and Series

- If two terms of a geometric sequence are $a_2 = -4$ and $a_6 = -1024$, write rule for the n^{th} term.

12.3 Analyze Geometric Sequences and Series

- Sum of geometric series
  - $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$
- Find the sum of the first 10 terms of
  - 4 + 2 + 1 + ½ + ⋯

12.3 Analyze Geometric Sequences and Series

- Find n such that $S_n = 31/4$
  - 4 + 2 + 1 + ½ + ⋯

12.4 Find the Sums of Infinite Geometric Series

- Sum of an infinite geometric series
  - $S = \frac{a_1}{1-r}$
- | r | < 1
  - If | r | > 1, then no sum (∞)

12.4 Find the Sums of Infinite Geometric Series

- Find the sum

$$\sum_{i=1}^{\infty} 2(0.1)^{i-1}$$

- $12 + 4 + \frac{4}{3} + \frac{4}{9} + \cdots$

12.4 Find the Sums of Infinite Geometric Series

- An infinite geometric series has $a_1 = 5$ has sum of 27/5. Find the common ratio.

12.4 Find the Sums of Infinite Geometric Series

- Write 0.27272727… as a fraction.

12.4 Find the Sums of Infinite Geometric Series

- Write 0.416666666… as a fraction.

12.5 Use Recursive Rules with Sequences and Functions

- Explicit Rule
  - Gives the n^{th} term directly
  - $a_n = 2 + 4n$
- Recursive Rule
  - Each term is found by knowing the previous term
  - $a_1 = 6$; $a_n = a_{n-1} + 4$

12.5 Use Recursive Rules with Sequences and Functions

- Write the first 5 terms
  - $a_1 = 1$, $a_n = (a_{n-1})^2 + 1$
- $a_1 = 2$, $a_2 = 2$, $a_n = a_{n-2} - a_{n-1}$

12.5 Use Recursive Rules with Sequences and Functions

- Write the rules for the arithmetic sequence where $a_1 = 15$ and $d = 5$.
  - Explicit
  - Recursive

12.5 Use Recursive Rules with Sequences and Functions

- Write the rule for the geometric sequence where $a_1 = 4$ and $r = 0.2$
  - Explicit
  - Recursive

12.5 Use Recursive Rules with Sequences and Functions

- Write a recursive rule for
  - 1, 1, 4, 10, 28, 76, …
- 1, 2, 2, 4, 8, 32, …